Interactions of x-rays with matter – introductory comments and insights (Part 2)

Synchrotrons and x-ray free-electron lasers

Techniques and applications

École polytechnique fédérale de Lausanne

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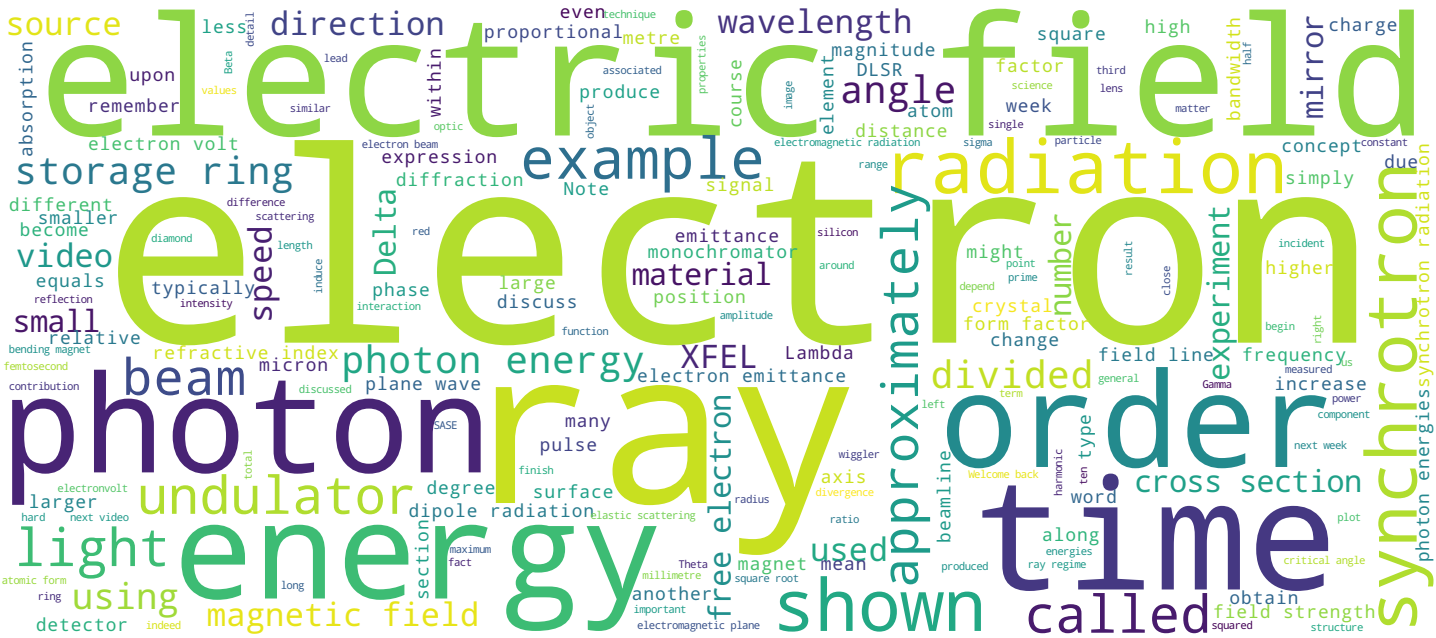


Search MOOC

Video

EPFL

Contents and objectives of this video

- Electric forces
- Response of a free electron to incident EM radiation
- Dipole radiation
  - Everyday life
  - Synchrotrons
- Size of an electron – the Thomson length

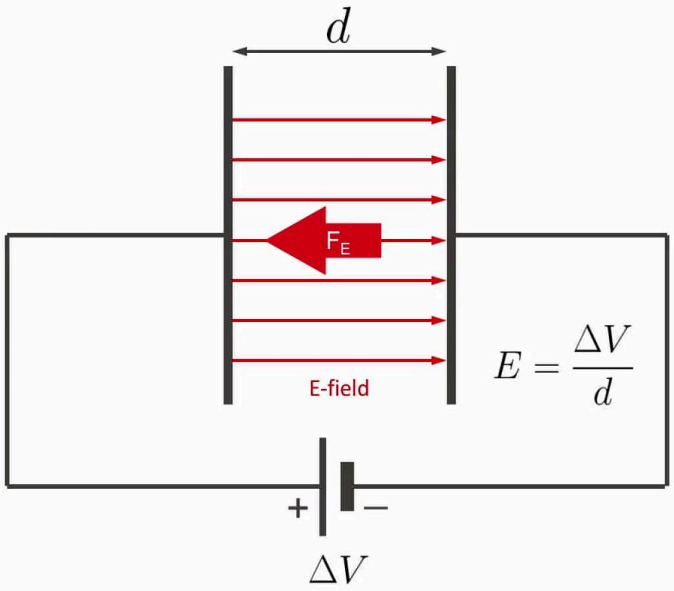
In this video we consider the forces on a charged particle, particularly an electron, imparted by an electric field, and thus how electrons respond to the oscillating electric field of electromagnetic radiation, which will lead us to the concept of dipole radiation. We finish by introducing the concept of the Thomson scattering length.

Notes

Summary

0m 05s

Electric fields



$$E = \frac{\Delta V}{d}$$

$$F_E = qE$$

$$F_E = -eE$$

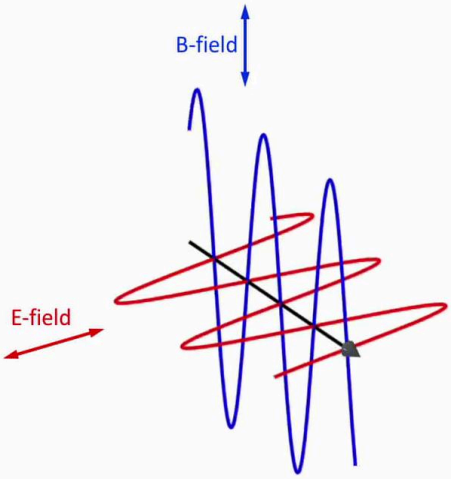
Electric fields are created by electric charges or by time-varying magnetic fields. Let us consider the electric field E generated by two conducting parallel plates of different electrical potential, Delta V. They are separated by a distance D. The field E is simply Delta V divided by D, and has electric field lines with a positive direction from the more positive plate to more negative potential plate. The force imparted on a particle of charge Q is FE is equal to qE. Note that, because an electron has a negative charge, Q equal to minus E, the electric force imparted on it is in the anti-parallel direction to that of the electric field.

Notes

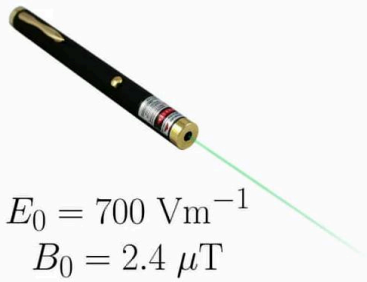
Summary

0m 29s

Plane-wave electromagnetic radiation



$$B_0\,[\mathrm{T}] = \frac{E_0\,[\mathrm{V\,m^{-1}}]}{c\,[\mathrm{m\,s^{-1}}]}$$



$E_0 = 700\ \mathrm{Vm^{-1}}$
$B_0 = 2.4\ \mu\mathrm{T}$

Transverse E- and B-fields ⇒ displacements perpendicular to direction of propagation

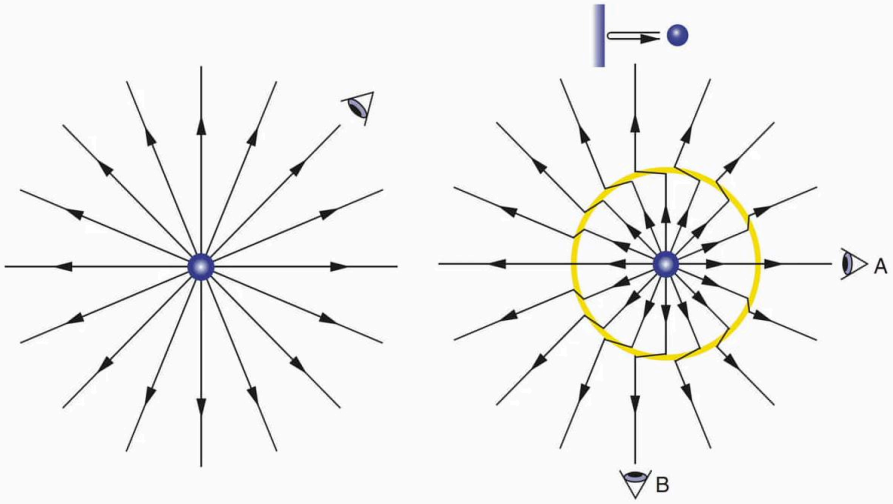
An electromagnetic plane wave has two interrelated fields: an oscillating electric field E and, perpendicular to this, a magnetic field B, oscillating at the same frequency and in phase with the electric field, as shown here. The fields are transverse— that is, the amplitudes lie perpendicular to the direction of propagation of the plane wave. Light is thus a transverse wave. This is an important fact to remember in the following discussion of dipole radiation. The ratio of the magnitudes of the electric and magnetic fields is fixed and given by: B in tesla is equal to the electric field strength in volts per metre, divided by C, the speed of light in metres per second. For example, it can be calculated that the electric field amplitude of a five milliwatt laser pointer with a three millimetre diameter beam is approximately 700 volts per metre. The magnetic field strength is only approximately 2.4 microtesla, an order of magnitude smaller than the typical values of the earth's magnetic field. We discuss this more in the context of generating synchrotron radiation in next week's videos.

Notes

Summary

1m 23s

Electrostatic fields



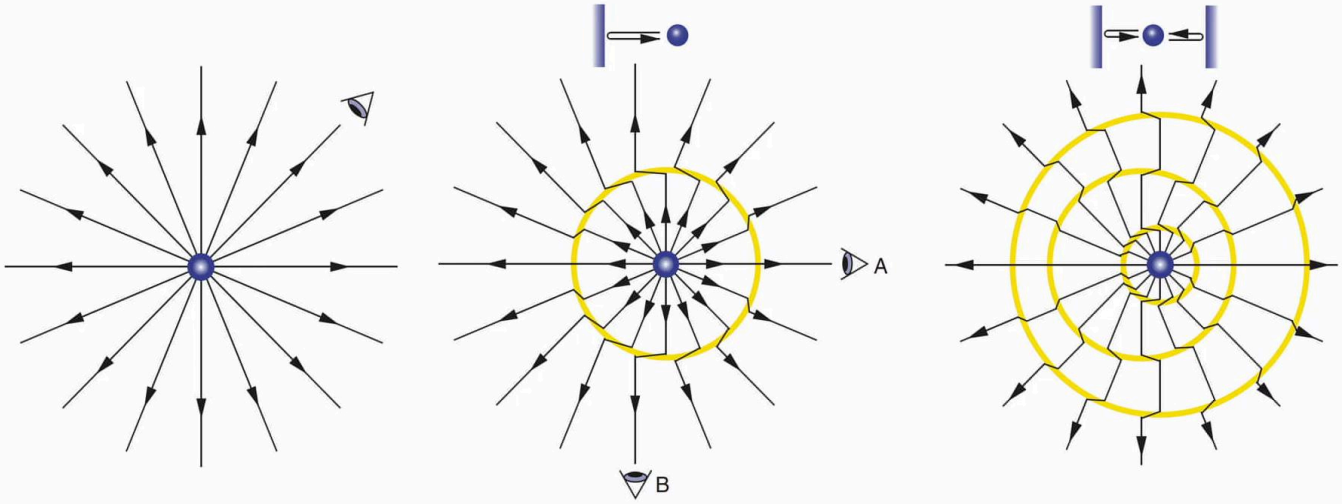
We now consider the electrostatic field generated by a point charge, such as a free electron, and how this changes if it is excited by an incident electromagnetic plane wave, such as a synchrotron beam. The electric field lines propagating at the speed of light from an isolated point-like charge all move radially outwards. Anyone observing the electron will, by geometric definition, be looking along an electric field line, and therefore, see no transverse component of the electric field. Seeing no transverse component means seeing no light because light is a transverse wave, as we have just stated. The electron thus emits no radiation. This is true either for a stationary electron or an electron moving with a constant velocity. Now let us imagine a situation where the electron bounces off a wall. This induces a sudden shift in the apparent origin of the electron, which manifests itself as a kink in the electric field lines that propagates out from the disturbed electron at the speed of light. Note that at position A an observer sees no lateral displacement, hence no radiation, while at position B a second observer registers a maximum kink amplitude, and thus a maximum in light strength.

Notes

Summary

2m 48s

Electrostatic fields



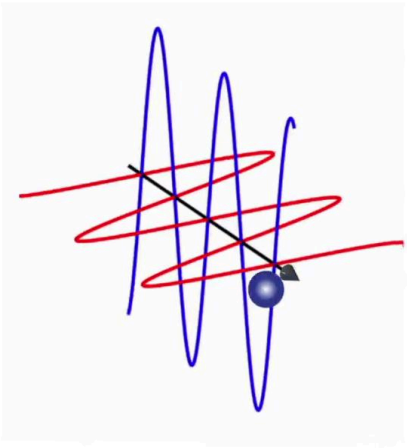
If we extend this thought experiment to the electron repeatedly bouncing off opposite walls, an observer anywhere other than along the axis of the electron motion will see repeated pulses of electromagnetic radiation separated by time intervals equal to the wall separation divided by the speed of the electron.

Notes

Summary

4m 20s

Driving an electron with electromagnetic radiation



$$F_E = -eE$$
$$= -eE_0 \sin \omega t$$

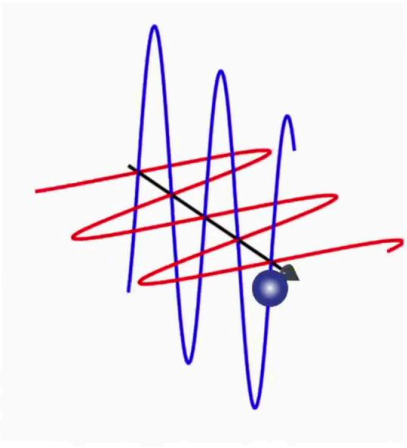
The case of an electron bouncing off opposite walls is similar to the case of an electron being driven to and fro by the electric field of an electromagnetic plane wave. Note that, in this cartoon, the electron moves exactly out of phase with the electric field, due to the electron having a negative charge.

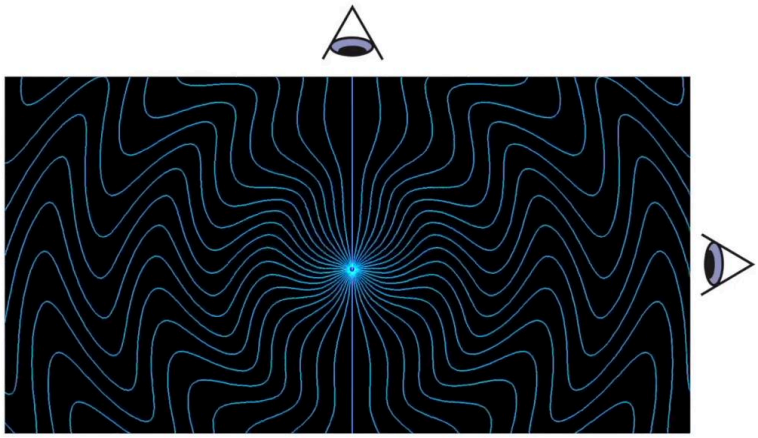
Notes

Summary

4m 43s

Driving an electron with electromagnetic radiation





http://www.shintakelab.com/en/enEducationalSoft.htm

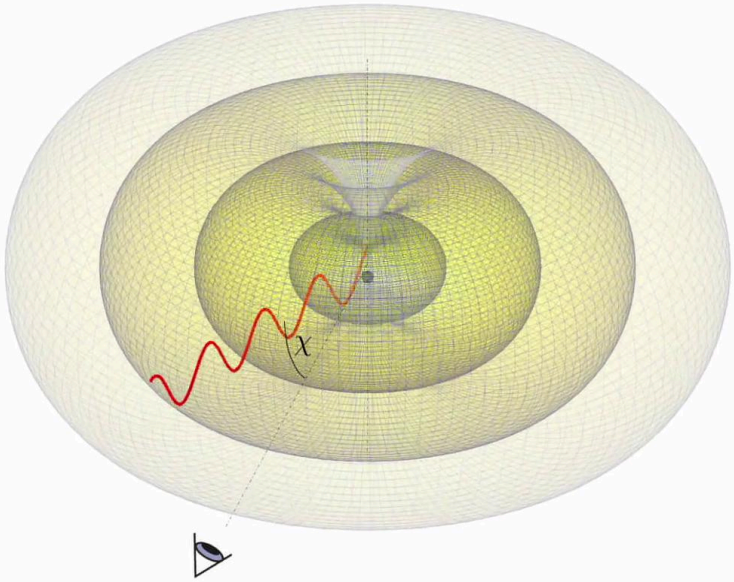
The electric field generated by the electron propagates outwards spherically as a sinusoidal field. Again, the amplitude exactly along the axis of the driving electric field is zero, while in the plane perpendicular to this it is largest.

Notes

Summary

5m 06s

Dipole radiation



$$I(\chi) = E_0^2 \cos^2 \chi$$

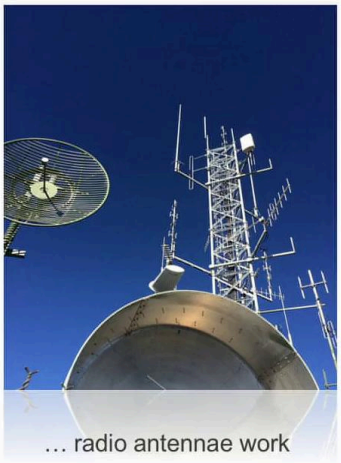
This scattered radiation from a sinusoidally accelerated free electron is called dipole radiation, or hertzian dipole radiation, to be precise, and is shown here in 3D. The dependence of the radiation intensity on Kai, the angle of observation relative to the plane perpendicular to the electric field, is given by I is equal to E zero squared cos squared Kai.

Notes

Summary

5m 27s

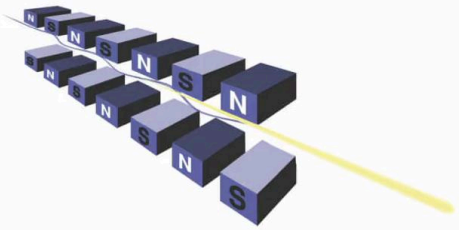
That's why...



… radio antennae work

Official White House photo © Pete Souza

… mirrors reflect



… synchrotrons produce x-rays

Dipole radiation is why radio antennae work. Radio waves washing over them induce the electrons in the metal of the antennae to oscillate in response, and generate an associated electric potential. You see yourself in a mirror because light scattered off you causes the electrons in the metal coated on the back of the glass to likewise oscillate and re-emit at the same frequencies. Lastly, electrons forced to execute sinusoidal paths as they move through the magnetic field of an undulator at a synchrotron also re-emit at the same frequency that they oscillate. The same frequency for them, that is, because, as we will see, relativistic effects mean that the emitted frequency which we as users of synchotron radiation perceive, is transformed into the X-ray regime. This is a subject we will tackle next week.

Notes

Summary

5m 57s

How big is a free electron?

- Assume it to be very small ($r_0 \ll \lambda$)
  - Thomson length r_0
    - Classical radius of electron
    - Derived by equating electrostatic and relativistic rest-mass energies

$$\frac{e^2}{4\pi\epsilon_0 r_0} = m_e c^2$$

$$r_0 = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = \boxed{2.82 \times 10^{-5}\ \text{Å}}$$

- Obtain free-electron cross-section by integrating dipole radiation over polar coordinates ϕ and χ. Leads to

$$\boxed{\sigma_T = \left(\frac{8\pi}{3}\right) r_0^2 = 0.665\ \text{b}}$$

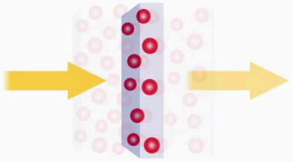
So, how big is a free electron? The Thomson length r zero is the classical radius of an electron and is derived by equating an electron's electrostatic energy at its surface equal to e squared divided by four Pi Epsilon zero r zero to its rest-mass energy m e c squared. From this we obtain r zero is equal to 2.82 times 10 to the minus five Å. The cross section of a free electron is obtained by integrating its dipole response over both polar coordinates, which yields an elastic scattering cross section, or Thomson cross section, of eight Pi divided by three times r zero squared, which in turn turns out to be close to two-thirds of a [inaudible 00:07:50].

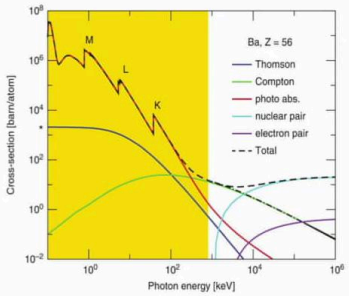
Notes

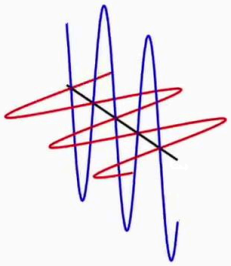
Summary

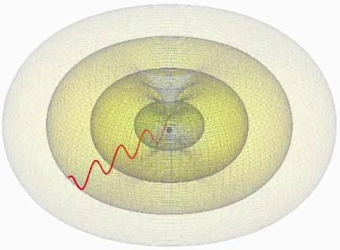
6m 57s

Summary of this section









Summarising this first section of week two, we discussed the concept of the cross sections for interactions in general, and briefly reviewed the properties and typical magnitudes of the three most important interactions of synchrotron radiation with matter, namely photo absorption, elastic or Thomson scattering, and Contin inelastic scattering. We then considered the forces imparted on charged particles by an electric field, and how a free electron responds to an incident electromagnetic plane wave to produce so-called dipole radiation. In the next section, we take a deeper look into elastic scattering, which will lead to the concept of the atomic form factor. This property describes both elastic scattering and photo absorption, from which we derive in the next [inaudible 00:08:44] section, expressions for the complex refractive index, and from this, a quantitative description of refraction, reflection, and absorption.

Notes

Summary

7m 51s